WEEKLY TEST TYJ-02 TEST 22 RAJPUR ROAD SOLUTION Date 12-01-2020

[PHYSICS]

1. **(a)** Time required for a point to move from maximum displacement to zero displacement is

$$t = \frac{T}{4} = \frac{1}{4n}$$

$$\Rightarrow \quad n = \frac{1}{4t} = \frac{1}{4 \times 0.170} = 1.47 \text{ Hz}$$

2. **(b)** Since the point $x = 0$ is a node and reflection is taking place from point $x = 0$. This means that reflection must be taking place from the fixed end and hence the reflected ray must suffer an additional phase change of π or a path change of $\lambda/2$

So, if $y_{\text{incident}} = a\cos(kx - \omega t)$

$$\Rightarrow \quad y_{\text{reflected}} = a\cos(-kx - \omega t + \pi)$$

$$= -a\cos(\omega t + kx)$$

3. **(c)** Critical hearing frequency for a person is 20,000 Hz.

If a closed pipe vibration in N^{th} mode then frequency of vibration

$$n = \frac{(2N-1)v}{4l} = (2N-1)n_1$$

(where n_1 = fundamental frequency of vibration)

Hence 20,000 = $(2N - 1) \times 1500 \Rightarrow N = 7.1 \approx 7$

Maximum possible harmonics obtained are

1,3,5,7,9,11,13

Hence, man can hear up to 13^{th} harmonic

$= 7 - 1 = 6$

So, number of overtones heard = 6

4. **(d)** Path difference $(\Delta x) = 50 \text{ cm} = \frac{1}{2}\text{m}$

$\therefore$ Phase difference $\Delta\phi = \frac{2\pi}{\lambda}$

$$\Delta x \Rightarrow \phi = \frac{2\pi}{1} \times \frac{1}{2} = \pi$$

Total phase difference $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$$\Rightarrow \quad A = \sqrt{a^2 + a^2 + 2a^2 \cos\left(\frac{2\pi}{3}\right)}a$$

5. **(a)** In first overtone mode, $l = \frac{3\lambda}{4}$

$$\therefore \quad \frac{\lambda}{4} = \frac{l}{3} = \frac{1.2}{3} = 0.4 \text{ m}$$

Pressure variation will be maximum at displacement nodes, i.e., at 0.4 m from the open end.

6. **(c)** The frequency of A, $n_A = n + \frac{2}{100}n$

and the frequency of B, $n_B = n - \frac{3}{100}n$

According to question, $n_A - n_B = 6$

$$\therefore \quad \left(n + \frac{2}{100}n\right) - \left(n - \frac{3}{100}n\right) = 6$$

or $\quad \frac{5}{100}n = 6 \quad \Rightarrow \quad n = \frac{600}{5} = 120 \text{ Hz}$

The frequency of A

$$n_A = \left(n + \frac{2}{100}n\right) = 120 + \frac{2}{100} \times 120$$

$$= 122.4 \text{ Hz}$$

7. **(a, c)** $v_{\max} = a\omega = \frac{v}{10} = \frac{10}{10} = 1 \text{ m/s}$

$$\Rightarrow \quad a\omega = a \times 2\pi n = 1$$

$$\Rightarrow \quad n = \frac{10^3}{2\pi} \qquad (\because a = 10^{-3} \text{ m})$$

Since $v = n\lambda \Rightarrow \lambda = \frac{v}{n} = \frac{10}{10^3/2\pi} = 2\pi \times 10^{-2} \text{ m}$

8. **(b)** $n=\frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow n_1l_1=n_2l_2=n_3l_3=k$

$$l_1+l_2+l_3=l \Rightarrow \frac{k}{n_1}+\frac{k}{n_2}+\frac{k}{n_3}=\frac{k}{n}$$

$$\Rightarrow \quad \frac{1}{n}=\frac{1}{n_1}+\frac{1}{n_2}+\frac{1}{n_3}+\cdots$$

9. **(d)** $<v>=\frac{v_1+v_2}{2}=\frac{\alpha\sqrt{T_1}+\alpha\sqrt{T_2}}{2}$

$$\Rightarrow \quad \text{Time taken } = \frac{2l}{\alpha(\sqrt{T_1}+\sqrt{T_2})}$$

Alternate Solution:

$$\frac{dx}{dt}=V=\alpha\sqrt{T_1+\left(\frac{T_2-T_1}{l}\right)x}$$

$$\int_{x=0}^{x=l}\frac{dx}{\sqrt{T_1+\left(\frac{T_2-T_1}{l}\right)x}}=\int_0^t \alpha dt$$

on solving we get $t=\frac{2l}{\alpha(\sqrt{T_1}+\sqrt{T_2})}$

10. **(c)** At $t=0$, $y=10\sin 2\pi\left(\frac{50x}{22}\right)$

Change in pressure will be maximum at $y=0$

$$y=0 \text{ at } \frac{(2\pi)(50x)}{22}=0, \pi, 2\pi, 3\pi, \ldots 100x\pi$$

$$=(3\pi)(22)$$

or $\quad x=0.66$ m

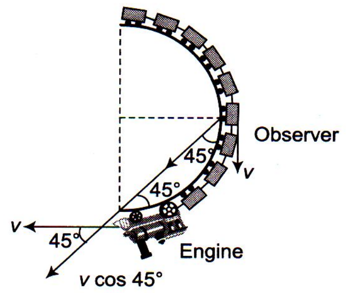
11. **(c)** Force closed pipe, $f=\frac{nV}{4\ell}$, $n=1, 3, 5...$

$$f_1=\frac{V}{4\ell}=\frac{330}{(4)(93.75/100)}=88 \text{ Hz}$$

$$f_2=\frac{3V}{4\ell}=\frac{(3)(330)}{(4)(93.75/100)}=264 \text{ Hz}$$

Required $f=264$ Hz

12. **(c)** The situation is shown in the fig. Both the source (engine) and the observer (Person in the middle of the train) have the same speed, but their direction of motion is right angles to each other. The component of velocity of observer towards source is $v \cos 45°$ and that of source along the time joining the observer and source is also $v \cos 45°$. There is number relative motion between them, so there is no change in frequency heard. So frequency heard is 200 Hz.



13. (a) When the train is approaching the stationary observer frequency heard by the observer $n' = \frac{v + v_0}{v} n$

when the train is moving away from the observer then frequency heard by the observer $n'' = \frac{v - v_0}{v} n$

it is clear that n' and n'' are constant and independent of time. Also and $n' > n''$.

14. (b) Equation of A, B, C and D are

$y_A = A \sin \omega t$, $y_B = A \sin(\omega t + \pi/2)$

$y_C = A \sin(\omega t - \pi/2)$, $y_D = A \sin(\omega t - \pi)$

It is clear that wave C lags behind by a phase angle of $\pi/2$ and the wave B is ahead by a phase angle at $\pi/2$.

15. (c) The particle velocity is maximum at B and is given by

$\frac{dy}{dt} = (v_p)_{\max} = \omega A$

Also wave velocity is $\frac{dx}{dt} = v = \frac{\omega}{k}$

So slope $\frac{dy}{dx} = \frac{(v_p)_{\max}}{v} = kA$

16. (d) Given equation $y = y_0 \sin(\omega t - \phi)$

at $t = 0$, $y = -y_0 \sin \phi$

this is the case with curve marked D.

17. (c) We know frequency $n = \frac{p}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} \Rightarrow n \propto \frac{1}{\sqrt{\rho}}$

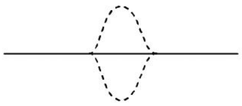
i.e., graph between n and $\sqrt{\rho}$ will be hyperbola.

18. (c) Energy density $(E) = \frac{I}{v} = 2\pi^2 \rho n^2 A^2$

$v_{max} = \omega A = 2\pi n A \Rightarrow E \propto (v_{max})^2$

i.e., graph between E and v_{max} will be a parabola symmetrical about E axis.

19. (c) After two seconds each wave travel a distance of $2.5 \times 2 = 5$ *cm i.e.* the two pulses will meet in mutually opposite phase and hence the amplitude of resultant will be zero.



20. (c) $n_Q = 341 \pm 3 = 344 Hz$ or $338 Hz$

on waxing Q, the number of beats decreases hence $n_Q = 344 Hz$

[CHEMISTRY]

21. (b)

Element	Na	K
IE_1	496	419
IE_2	4562	3051

Sodium has higher I.E. because of smaller atomic size.

22. (c) Alkali metals are highly reactive metals. They react with

Alcohol – $2C_2H_5OH + 2K \rightarrow 2C_2H_5OK + H_2$

Water – $2K + 2H_2O \rightarrow 2KOH + H_2$

Ammonia – $K + (x+y)NH_3 \rightarrow \underset{\text{Ammoniated cation}}{[K(NH_3)_x]^+} + \underset{\text{Ammoniated electron}}{[e(NH_3)_y]^-}$

But they do not react with kerosene.

23. (b) After removal of an electron the effective nuclear charge per electron increases hence the size decreases.

24. (a) Alkali metals valence shell configuration $= ns^1$

25. (b)

Element –	Li	Na	K	Rb	Cs
Ionic radius – (pm)	76	102	138	152	167

as the atomic no. increases the no. of shells increases hence, atomic radius increases.

26. (c) On moving down the group electropositive character increases.

27. (c)

28. (d)

Element –	Li	Na	K	Rb
Atomic radius (pm) –	152	186	227	248

29. (b) Li is much softer than the other group I metals. Actually Li is harder then other alkali metals

30. (a) $Cu^{+2} + 2e^- \rightarrow Cu$, $E^o = +0.34\ V$

$Mg^{+2} + 2e^- \rightarrow Mg$, $E^o = -2.37\ V$

$Na^+ + e^- \rightarrow Na$, $E^o = -2.71\ V$

31. (d) Anhydrous form of Na_2CO_3 does not decompose on heating even to redness. It is a amorphous powder called soda ash.

32. (a)

33. (b) Although lattice energy of $LiCl$ higher than $NaCl$ but $LiCl$ is covalent in nature and $NaCl$ ionic there after, the melting point decreases as we move $NaCl$ because the lattice energy decreases as a size of alkali metal atom increases (lattice energy $\propto$ melting point of alkali metal halide)

34. (d)

35. (d)

36. (b) $2NaCl + 2H_2O \xrightarrow{\text{Electrolysis}} 2NaOH + \underset{\text{Anode}}{Cl_2} + \underset{\text{Cathode}}{H_2}$

37. (d) $CaSO_4.\frac{1}{2}H_2O$ or $(CaSO_4)_2.H_2O$

38. (d) Setting of plaster of paris is exothermic process

$CaSO_4.\frac{1}{2}H_2O \xrightarrow{H_2O} \underset{\text{Orthorhombic}}{CaSO_4.2H_2O}$

$\xrightarrow{\text{Hardening}} \underset{\text{Mono orthorhombic Gypsum}}{CaSO_4.2H_2O}$

The setting is due to formation of another hydrate

39. (d)

40. (a) $MgCO_3 \xrightarrow{\text{Heat}} MgO + CO_2$

[MATHEMATICS]

41. (d) $a = 4$, vertex $= (0,0)$, focus $= (0,-4)$.

42. (c) Vertex $= (2,0) \Rightarrow$ focus is $(2+2, 0) = (4, 0)$.

43. (c) The point $(-3,2)$ will satisfy the equation $y^2 = 4ax$

$\Rightarrow 4 = -12a \Rightarrow 4a = -\frac{4}{3} = \frac{4}{3}$, (Taking positive sign).

44. (a) Given equation is $x^2 = -8ay$. Here $A = 2a$

Focus of parabola $(0, -A)$ *i.e.* $(0, -2a)$

Directrix $y = A$ *i.e.*, $y = 2a$.

45. (a) $(y-2)^2 = -4x + 4 \Rightarrow (y-2)^2 = -4(x-1)$

Vertex is $(1,2)$ and focus $= (0,2)$.

46. (a) $(x+2)^2 = -2y + 7 + 4 \Rightarrow (x+2)^2 = -2\left(y - \frac{11}{2}\right)$

Hence vertex is $\left(-2, \frac{11}{2}\right)$.

47. (a) Since $9y^2 - 16x - 12y - 57 = 0$

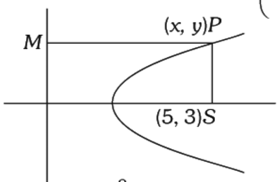
$\Rightarrow \left(y - \frac{2}{3}\right)^2 = \frac{16}{9}\left(x + \frac{61}{16}\right)$

Put $y - \frac{2}{3} = Y$ and $x + \frac{61}{16} = X \Rightarrow Y^2 = 4\left(\frac{4}{9}\right)X$

Axis of this parabola is $Y = 0 \Rightarrow y - \frac{2}{3} = 0 \Rightarrow 3y = 2$.

48. (b) Always eccentricity of parabola is $e = 1$.

49. (a) $PM^2 = PS^2 \Rightarrow (x-5)^2 + (y-3)^2 = \left(\frac{3x - 4y + 1}{\sqrt{9+16}}\right)^2$



$\Rightarrow 25(x^2 + 25 - 10x + y^2 + 9 - 6x)$

$= 9x^2 + 16y^2 + 1 - 12xy + 6x - 8y - 12xy$

$\Rightarrow 16x^2 + 9y^2 - 256x - 142y + 24xy + 849 = 0$

$\Rightarrow (4x + 3y)^2 - 256x - 142y + 849 = 0.$

50. (c) According to the condition, $\frac{2a}{e} = 6ae \Rightarrow e = \frac{1}{\sqrt{3}}$.

51. (b) $\frac{x^2}{(48/3)} + \frac{y^2}{(48/4)} = 1$

$a^2 = 16, b^2 = 12 \Rightarrow e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{2}$

Distance is $2ae = 2 \cdot 4 \cdot \frac{1}{2} = 4$.

52. (d) $e = \frac{1}{\sqrt{2}}$; Latus rectum $= \frac{2b^2}{a} = \frac{2a^2}{a}\left(1 - \frac{1}{2}\right) = a$

i.e., semi-major axis.

53. (c) $\frac{x^2}{\frac{112}{16}} + \frac{y^2}{\frac{112}{7}} = 1$. Therefore, $e = \sqrt{1 - \frac{112}{16} \cdot \frac{7}{112}} = \frac{3}{4}$.

54. (a) Centre $(0,0)$, focus $(0,3)$, $b = 5$

Focus $(0,3) \Rightarrow be = 3 \Rightarrow e = 3/5 \Rightarrow a = b\sqrt{1 - e^2} = 4$

Hence the required equation is $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

55. (a) $\frac{2b^2}{a} = 8$ and $\frac{3}{\sqrt{5}} = \sqrt{1 + \frac{b^2}{a^2}}$ or $\frac{4}{5} = \frac{b^2}{a^2}$

$\Rightarrow a = 5\ ,\ b = 2\sqrt{5}$.

Hence the required equation of hyperbola is $\frac{x^2}{25} - \frac{y^2}{20} = 1 \Rightarrow 4x^2 - 5y^2 = 100$.

56. (a) Conjugate axis is 5 and distance between foci $= 13 \Rightarrow 2b = 5$ and $2ae = 13$.
Now, also we know for hyperbola

$b^2 = a^2(e^2 - 1) \Rightarrow \frac{25}{4} = \frac{(13)^2}{4e^2}(e^2 - 1)$

$\Rightarrow \frac{25}{4} = \frac{169}{4} - \frac{169}{4e^2}$ or $e^2 = \frac{169}{144} \Rightarrow e = \frac{13}{12}$

or $a = 6, b = \frac{5}{2}$ or hyperbola is $\frac{x^2}{36} - \frac{y^2}{25/4} = 1$

$\Rightarrow 25x^2 - 144y^2 = 900$.

57. (c) Foci $(0, \pm 4) \equiv (0, \pm be) \Rightarrow be = 4$

Vertices $(0, \pm 2) \equiv (0, \pm b) \Rightarrow b = 2 \Rightarrow a = 2\sqrt{3}$

Hence equation is $\frac{-x^2}{(2\sqrt{3})^2} + \frac{y^2}{(2)^2} = 1$ or $\frac{y^2}{4} - \frac{x^2}{12} = 1$.

58. (d) Given conic is $\frac{x^2}{(1)^2} - \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1$

$\therefore b^2 = a^2(e^2 - 1) \Rightarrow \frac{1}{4} + 1 = e^2 \Rightarrow e = \frac{\sqrt{5}}{2}$.

59. (c) The equation of hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Now $b^2 = a^2(e^2 - 1) \Rightarrow e = \frac{5}{4}$

Hence foci are $(\pm ae, 0) \Rightarrow \left(\pm 4.\frac{5}{4}, 0\right)$ *i.e.*, $(\pm 5, 0)$.

60. (a) $a = 4, b = 3 \Rightarrow \frac{9}{16} = (e^2 - 1) \Rightarrow e = \frac{5}{4}$